B.Sc. Part—III Semester—VI Examination

MATHEMATICS

(Special Theory of Relativity)

Paper—XII (Optional)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory, attempt it once only.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative :

(1) If ϕ is the scalar potential and $\overline{A}$ is the vector potential then the electric field $\overline{E}$ is :

(a) $\overline{E} = \text{grad}\ \phi + \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$ (b) $\overline{E} = -\text{grad}\ \phi + \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$

(c) $\overline{E} = -\text{grad}\ \phi - \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$ (d) $\overline{E} = \text{grad}\ \phi - \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$

(2) The order of outer product is the _____ of the order of the tensors.

(a) Sum (b) Difference

(c) Product (d) None of these

(3) Newton's fundamental equations of motion are invariant under :

(a) Lorentz transformation (b) Galilean transformation

(c) General Lorentz transformation (d) None of these

(4) In relativistic addition law for velocities when $C \to \infty$. Then :

(a) $u' = v - u$ (b) $u' = u - v$

(c) $u' = u + v$ (d) None of these

(5) The reference system is said to be an inertial system if :

(a) Newton's first law of motion valid (b) Newton's second law of motion valid

(c) Newton's third law of motion valid (d) None of these

(6) The electromagnetic field tensor (or Maxwell tensor) F_{ij} is defined as :

(a) $F_{ij} = \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ (b) $F_{ij} = \frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j}$

(c) $F_{ij} = \frac{\partial A_i}{\partial x^j} + \frac{\partial A_j}{\partial x^i}$ (d) $F_{ij} = \frac{\partial A_j}{\partial x^i} + \frac{\partial A_i}{\partial x^i}$

(7) In special relativity, the simultaneity is :

(a) Constant (b) Relative

(c) Absolute (d) None of these

(8) If $\overline{A}$ is a vector potential then the magnetic field is given by :

(a) $\overline{H} = \text{div}\ \overline{A}$

(b) $\overline{H} = \text{curl}\ \overline{A}$

(c) $\overline{H} = \nabla\phi \times A$

(d) None of these

(9) Sum of two tensors A_k^{ij} and B_k^{ij} is a mixed tensor of order :

(a) 6

(b) 3

(c) 9

(d) None of these

(10) The mass of a moving particle $m = \dfrac{m_0}{\sqrt{1 - \dfrac{u^2}{c^2}}}$ is called :

(a) Equivalent mass of a particle

(b) Relativistic mass of a particle

(c) Rest mass of a particle

(d) None of these 1×10=10

UNIT—I

2. (a) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 5

(b) Prove that Newton's fundamental equations of motion are invariant under the Galilean transformation. 5

3. (c) Prove that $\nabla^2 - \dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformation. 5

(d) Discuss the Geometrical interpretation of Lorentz transformation. 5

UNIT—II

4. (a) If $\overline{u}$ and $\overline{u}'$ be the velocities of a particle in the inertial systems s and s′ respectively where s′ is moving with velocity v relative to s along xx′ axis then show that :

$$\tan\theta' = \frac{\sin\theta\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}}{\cos\theta - \dfrac{v}{u}}$$

and

$$u' = \frac{u^2\left[1 - 2\dfrac{v}{u}\cos\theta + \dfrac{v^2}{u^2} - \dfrac{v^2}{c^2}\sin^2\theta\right]}{\left(1 - \dfrac{uv}{c^2}\cos\theta\right)^2}$$

where θ and θ′ are the angles made by u and u′ with the x-axis respectively. 5

(b) Obtain the transformation of Lorentz contraction factor. 5

5. (c) Write short notes on :

(i) Time dilation

(ii) Length contraction. 2+3

(d) Obtain the transformation for the velocity of a particle under special Lorentz transformation. 5

UNIT—III

6. (a) Obtain the transformation of the components of a symmetrical four tensor T'^{12}, T'^{23} under the LT. 5

(b) The metric of the space-time geometry of special relativity in frame S is given by

$$ds^2 = -(dx')^2 - (dx^2)^2 - (dx^3)^2 + (dx^4)^2$$

Show that ds^2 is invariant under Lorentz transformation. 5

7. (c) Define :

(i) Time-like

(ii) Space-like

(iii) Light-like

(iv) Covariant tensor of order 2

(v) Signature of the metric. 5

(d) Define four vectors, show that the square of the length of a four vector is invariant under Lorentz Transformation. 5

UNIT—IV

8. (a) Deduce Einstein's Mass Energy equivalence relation. 5

(b) Prove that the four velocity and four acceleration are orthogonal to each other. 5

9. (c) Derive the relativistic equation :

$$m = \alpha\left(1 + \frac{vux'}{c^2}\right)m' \text{ for mass where } \alpha = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \qquad 5$$

(d) Define four momentum vector $\dot{p}$. Show that the quantity $p^2 - \frac{E^2}{c^2}$ is an invariant whose numerical value is $-m_0^2 c^2$. 5

UNIT—V

10. (a) Prove that the set of Maxwell's equations $\operatorname{div} \overline{H} = 0$ and $\operatorname{curl} \overline{E} = -\frac{1}{c}\frac{\partial \overline{H}}{\partial t}$ can be written using electromagnetic field tensor as :

$$\frac{\partial F_{ij}}{\partial x^{k}} + \frac{\partial F_{jk}}{\partial x^{i}} \quad \frac{\partial F_{ki}}{\partial x^{j}} = 0$$ 5

(b) Show that the Lorentz force acting on a particle of charge 'e' is given by :

$$\overline{F}_{L} = e\left(\overline{E} + \frac{1}{c}\overline{u} \times \overline{H}\right)$$ 5

11. (c) Define Current four vector. Show that $c^2e^2 - J^2$ is invariant and its value is $p_0^2c^2$.
5

(d) Obtain the transformation for electric and magnetic field strengths. 5